Lesson 9. Multiperiod Models

Example 1. Priceler manufactures sedans and wagons. The demand for each type of vehicle in the next three months is:

	Sedans	Wagons
Month 1	1100	600
Month 2	1500	700
Month 3	1200	500

Assume that the demand for both vehicles must be met exactly each month. Each sedan costs \$2000 to produce, and each wagon costs \$1500 to produce. Vehicles not sold in a given month can be held in inventory. To hold a vehicle in inventory from one month to the next costs \$150 per sedan and \$200 per wagon. During each month, at most 1500 vehicles can be produced. At the beginning of month 1, 200 sedans and 100 wagons are available. Formulate a linear program that can be used to minimize Priceler's costs during the next three months.

- First, let's write a linear program without sets and parameters, so we can understand the problem better.

DVs. $x_{s,1}$ = # sedans to produce in month 1
$x_{w,1}$ = # wagons to produce in month 1
$x_{s,2}, x_{s,3}, x_{w,2}, x_{w,3}$ defined similarly.

$y_{s,1}$ = # sedans to hold in inventory at the end of month 1
$y_{w,1}$ = # wagons to hold in inventory at the end of month 1
$y_{s,2}, y_{s,3}, y_{w,2}, y_{w,3}$ defined similarly.

$$\min \quad 2000(x_{s,1} + x_{s,2} + x_{s,3}) + 1500(x_{w,1} + x_{w,2} + x_{w,3}) + 150(y_{s,1} + y_{s,2} + y_{s,3}) + 200(y_{w,1} + y_{w,2} + y_{w,3}) \quad \text{(total cost)}$$

s.t.

$$\left.\begin{aligned} 200 + x_{s,1} &= 1100 + y_{s,1} \\ y_{s,1} + x_{s,2} &= 1500 + y_{s,2} \\ y_{s,2} + x_{s,3} &= 1200 + y_{s,3} \end{aligned}\right\} \text{(sedan balance)}$$

$$\left.\begin{aligned} x_{s,1} + x_{w,1} &\le 1500 \\ x_{s,2} + x_{w,2} &\le 1500 \\ x_{s,3} + x_{w,3} &\le 1500 \end{aligned}\right. \quad \text{(monthly prod. capacity)}$$

$$\left.\begin{aligned} 100 + x_{w,1} &= 600 + y_{w,1} \\ y_{w,1} + x_{w,2} &= 700 + y_{w,2} \\ y_{w,2} + x_{w,3} &= 500 + y_{w,3} \end{aligned}\right\} \text{(wagon balance)}$$

$$\left.\begin{aligned} x_{s,1}, x_{s,2}, x_{s,3}, x_{w,1}, x_{w,2}, x_{w,3} &\ge 0 \\ y_{s,1}, y_{s,2}, y_{s,3}, y_{w,1}, y_{w,2}, y_{w,3} &\ge 0 \end{aligned}\right\} \text{(nonnegativity)}$$

- Now, let's write a parameterized linear program.

Sets. V = set of vehicle types = $\{s, w\}$
T = set of months = $\{1, 2, 3\}$

Parameters.
p_i = unit production cost for vehicle i for $i \in V$
h_i = unit holding cost for vehicle i for $i \in V$
$d_{i,t}$ = demand for vehicle i in month t for $i \in V$ and $t \in T$
I_i = initial inventory for vehicle i for $i \in V$

DVs.
$x_{i,t}$ = # vehicle i to produce in month t for $i \in V$, $t \in T$
$y_{i,t}$ = # vehicle i to hold at the end of month t for $i \in V$, $t \in T \cup \{0\}$

"union"

$$\min \quad \sum_{i \in V} p_i \sum_{t \in T} x_{i,t} + \sum_{i \in V} h_i \sum_{t \in T} y_{i,t} \qquad \text{(total cost)}$$

s.t.

$$\sum_{i \in V} x_{i,t} \leq 1500 \quad \text{for } t \in T \qquad \text{(monthly prod. capacity)}$$

$$y_{i,0} = I_i \quad \text{for } i \in V \qquad \text{(initial inventory)}$$

$$y_{i,t-1} + x_{i,t} = d_{i,t} + y_{i,t} \quad \text{for } i \in V, t \in T \qquad \text{(balance)}$$

$$x_{i,t} \geq 0 \quad \text{for } i \in V, t \in T$$
$$y_{i,t} \geq 0 \quad \text{for } i \in V, t \in T \cup \{0\} \qquad \text{(nonnegativity)}$$

Example 2. During the next three months, the Bellman Company must meet the following demands for their line of advanced GPS navigation systems:

Month 1	Month 2	Month 3
1200	1400	2200

It takes 1 hour of labor to produce 1 GPS system. During each of the next three months, the following number of regular-time labor hours are available:

Month 1	Month 2	Month 3
1200	1300	1000

Each month, the company can require workers to put in up to 500 hours of overtime. Workers are only paid for the hours they work. A worker receives \$10 per hour for regular-time work and \$15 per hour for overtime work. GPS systems produced in a given month can be used to meet demand in that month, or put into a warehouse. Holding a GPS system in the warehouse from one month to the next costs \$2 per GPS system. Formulate a linear program that minimizes the total cost incurred in meeting the demands of the next three months.

Sets. T = set of months = $\{1, 2, 3\}$

Params. d_t = demand in month t for $t \in T$

c = unit cost for a GPS made w/ regular labor = 10

b = unit cost for a GPS made w/ overtime labor = 15

h = unit holding cost per GPS = 2

r_t = # GPS that can be made w/ regular-time labor in month t for $t \in T$

v = # GPS that can be made w/ overtime labor in each month = 500

DVs. x_t = # GPS produced by regular-time labor in month t for $t \in T$

y_t = # GPS produced by overtime labor in month t for $t \in T$

z_t = # GPS held from month $t \to t+1$ for $t \in T \cup \{0\}$

$$\min \quad c \sum_{t \in T} x_t + b \sum_{t \in T} y_t + h \sum_{t \in T} z_t \qquad \text{(total cost)}$$

s.t.

$$y_t \leq v \quad \text{for } t \in T \qquad \text{(overtime capacity)}$$

$$x_t \leq r_t \quad \text{for } t \in T \qquad \text{(regular time capacity)}$$

$$z_{t-1} + x_t + y_t = d_t + z_t \quad \text{for } t \in T \qquad \text{(balance)}$$

$$z_0 = 0 \qquad \text{(initial inventory)}$$

$$x_t \geq 0, \; y_t \geq 0 \quad \text{for } t \in T$$

$$z_t \geq 0 \quad \text{for } t \in T \cup \{0\} \qquad \text{(nonnegativity)}$$